Stability of Cylindrical Shell Panels of Modern Materials under Dynamic Loading

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INTRODUCTION AND FORMULATION OF THE PROBLEM

Dynamic loading has a significant effect on design life and efficiency of structures. In particular, for thin-walled shell structures used in aircraft construction, building construction, and other industrial fields, dynamic loading may lead to a loss of stability (significant sudden increase in deflection with a small change in the applied load). Frequently this leads to a sudden increase of stresses in the shell material and emergence of irreversible changes (appearance of microcracks and flow deformations).

The purpose of this work is to analyze the stability of some variants of shell structures made of modern orthotropic materials under dynamic loading.

THEORY AND METHODS

We will use a geometrically nonlinear variant of the mathematical model which also takes into account the orthotropy of the material and transverse shears (model of the Timoshenko type). The middle surface of the shell is taken as the coordinate surface. The x axis is directed along the generatrix of a cylindrical panel, the y axis along the directrix, and the z axis along the normal to the middle surface in the direction of the concavity (Fig. 1).

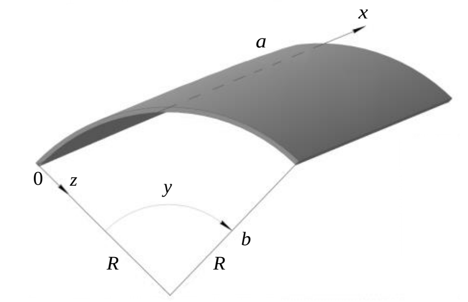

FIGURE 1. Schematic representation of a cylindrical panel

Taking into account geometric nonlinearity and transverse shears, the geometric relationships in the middle surface of a cylindrical panel will have the form:

$$\varepsilon_x = \frac{\partial U}{\partial x} + \frac{1}{2}\theta_1^2,\ \varepsilon_y = \frac{1}{R}\frac{\partial V}{\partial y} - \frac{1}{R}W + \frac{1}{2}\theta_2^2,\ \gamma_{xy} = \frac{\partial V}{\partial x} + \frac{1}{R}\frac{\partial U}{\partial y} + \theta_1\theta_2,\ \gamma_{xz} = k\ f(z)\left[\Psi_x - \theta_1\right],\ \gamma_{yz} = k\ f(z)\left[\Psi_y - \theta_2\right],\ \chi_1 = \frac{\partial \Psi_x}{\partial x},\ \chi_2 = \frac{1}{R}\frac{\partial \Psi_y}{\partial y},\ \chi_{12} = \frac{1}{2}\left(\frac{\partial \Psi_y}{\partial x} + \frac{1}{R}\frac{\partial \Psi_x}{\partial y}\right),\quad (1)$$

where $U = U(x,y,t), V = V(x,y,t), W = W(x,y,t)$ are unknown displacement functions, and $\Psi_x = \Psi_x(x,y,t),\ \Psi_y = \Psi_y(x,y,t)$ are unknown functions of the normal rotation angles in the xOz and yOz planes respectively; $\varepsilon_x, \varepsilon_y$ are the strain deformations along the coordinates x, y of the middle surface; $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$ are the shear deformations in the xOy, xOz, yOz planes respectively; $f(z)$ is a function characterizing the distribution of the shear deformations γ_{xz}, γ_{yz} by the shell thickness; $\chi_1, \chi_2, \chi_{12}$ are functions of the change of curvature and torsion; R is the radius; $k = 5/6$; and $\theta_1 = -\frac{\partial W}{\partial x},\quad \theta_2 = -\left(\frac{1}{R}\frac{\partial W}{\partial y} + \frac{1}{R}V\right)$. We introduce the dimensionless parameters:

$$\xi = \frac{x}{a},\ \eta = \frac{y}{b},\ \bar{\lambda} = \frac{a}{bR},\ \bar{U} = \frac{aU}{h^2},\ \bar{V} = \frac{bVR}{h^2},\ \bar{W} = \frac{W}{h},\ \bar{\Psi}_x = \frac{\Psi_x a}{h},\ \bar{\Psi}_y = \frac{\Psi_y bR}{h},\ \bar{t} = \frac{h}{a^2 A^2}\sqrt{\frac{E_1}{(1-\mu_{12}\mu_{21})\rho}}\cdot t,\ \bar{P} = \frac{a^4 q}{h^4 E_1},\ \bar{A} = \frac{a}{h},\ \bar{B} = \frac{bR}{h},\quad (2)$$

where ρ is material density; h is panel thickness; E_1, E_2 are elastic moduli; G_{12}, G_{13}, G_{23} are shear moduli; and μ_{12}, μ_{21} are Poisson's ratios. The total deformation energy of a shell structure can be written with the functional $I = \int_{\bar{t}_0}^{\bar{t}_1}\left(\bar{K} - \bar{E}_p\right)d\bar{t}$, (3), where $\bar{K}$ is the kinetic deformation energy of the system, and $\bar{E}_p$ is the functional of the static problem, equal to the difference in the potential deformation energy of the system and the work of external forces:

$$\bar{E}_p = \int_0^1\int_0^1\Big\{\varepsilon_x^2 + \bar{G}_2\varepsilon_y^2 + 2\mu_{21}\varepsilon_x\varepsilon_y + \bar{G}_{12}\bar{\lambda}^2\gamma_{xy}^2 + \bar{G}_{13}k\bar{A}^2\left(\bar{\Psi}_x - \bar{\theta}_1\right)^2 + \bar{G}_{23}k\bar{A}^2\bar{\lambda}^2\left(\bar{\Psi}_y - \bar{\theta}_2\right)^2 + \frac{1}{12}\left(\chi_1^2 + \bar{G}_2\chi_2^2 + 2\mu_{21}\chi_1\chi_2 + 4\bar{G}_{12}\bar{\lambda}^2\chi_{12}^2\right) - 2\left(1-\mu_{12}\mu_{21}\right)\bar{P}\bar{W}\Big\}\bar{A}\bar{B}\,d\xi\, d\eta,$$

$$\bar{K} = \frac{1}{\bar{A}^2}\int_0^1\int_0^1\left\{\left(\frac{\partial \bar{U}}{\partial \bar{t}}\right)^2 + \bar{\lambda}^2\left(\frac{\partial \bar{V}}{\partial \bar{t}}\right)^2 + \bar{A}^2\left(\frac{\partial \bar{W}}{\partial \bar{t}}\right)^2 + \frac{1}{12}\left[\left(\frac{\partial \bar{\Psi}_x}{\partial \bar{t}}\right)^2 + \bar{\lambda}^2\left(\frac{\partial \bar{\Psi}_y}{\partial \bar{t}}\right)^2\right]\right\}\bar{A}\bar{B}\,d\xi\, d\eta,\quad \bar{G}_2 = \frac{E_2}{E_1},\ \bar{G}_{12} = \frac{G_{12}(1-\mu_{12}\mu_{21})}{E_1},\ \bar{G}_{13} = \frac{G_{13}(1-\mu_{12}\mu_{21})}{E_1},\ \bar{G}_{23} = \frac{G_{23}(1-\mu_{12}\mu_{21})}{E_1}\quad (4)$$

According to the classical variation of the L.V. Kantorovich method, the required displacement functions and functions of the normal rotation angles are presented in the form

$$\bar{U}(\xi,\eta,\bar{t}) = \sum_{i=1}^{N}\bar{U}_i(\bar{t})Z1(i,\xi,\eta),\quad \bar{V}(\xi,\eta,\bar{t}) = \sum_{i=1}^{N}\bar{V}_i(\bar{t})Z2(i,\xi,\eta),\quad \bar{W}(\xi,\eta,\bar{t}) = \sum_{i=1}^{N}\bar{W}_i(\bar{t})Z3(i,\xi,\eta),\quad \bar{\Psi}_x(\xi,\eta,\bar{t}) = \sum_{i=1}^{N}\bar{\Psi}_{xi}(\bar{t})Z4(i,\xi,\eta),\quad \bar{\Psi}_y(\xi,\eta,\bar{t}) = \sum_{i=1}^{N}\bar{\Psi}_{y_i}(\bar{t})Z5(i,\xi,\eta),\quad (5)$$

where $\bar{U}_i - \bar{\Psi}_{y_i}$ are unknown functions of the variable $\bar{t}$, and $Z1 - Z5$ are known approximating functions that satisfy given boundary conditions. As a rule, these functions are different combinations of trigonometric functions with various arguments.

Then functions (5) are substituted into the functional of the total deformation energy of the shell (4). After calculating the integrals over variables ξ and η from known functions, the functional I becomes a one-dimensional functional of the functions $\bar{U}_i(\bar{t}) - \bar{\Psi}_{y_i}(\bar{t})$. Equations of motion, which are an ODE system, are obtained from the minimum conditions for this functional, $\delta I = 0$. Likewise, the system thus derived is called a multidimensional variant of the Euler–Lagrange equation:

$$\frac{d}{d\bar{t}}\frac{\partial\left(\bar{K} - \bar{E}_p\right)}{\partial \dot{X}_k(\bar{t})} - \frac{\partial\left(\bar{K} - \bar{E}_p\right)}{\partial X_k(\bar{t})} = 0,\quad k = 1, 2, \ldots, 5N,\quad (6),$$ where $X(\bar{t}) = \left(\bar{U}_i(\bar{t}), \bar{V}_i(\bar{t}), \bar{W}_i(\bar{t}), \bar{\Psi}_{xi}(\bar{t}), \bar{\Psi}_{y_i}(\bar{t})\right)^T,\quad i = 1,.., N,$ and the dot denotes the time derivative.

Since the derivatives of the required function with respect to variable $\bar{t}$ are contained only in the expression for the kinetic energy, and the functions themselves only in the expression for $\bar{E}_p$, then the following is true $\frac{d}{d\bar{t}}\frac{\partial \bar{K}}{\partial \dot{X}_k(\bar{t})} + \frac{\partial \bar{E}_p}{\partial X_k(\bar{t})} = 0,\ k = 1, 2, \ldots, 5N.$ (7)

Moreover, $\frac{\partial \bar{E}_p}{\partial X_k(\bar{t})} = 0,\ k = 1, 2, \ldots, 5N$ is a system of equations of a static problem [9]. The process of formulating system (7) was programmed in the analytical computing environment Maple 2016. The resulting system of ODE was solved numerically by the Rosenbrock method, which is effective in solving rigid systems.

NUMERICAL RESULTS

We consider isotropic and orthotropic cylindrical panels that have fixed-pin joints along the contour. The transverse load acting on the structure is uniformly distributed and linearly dependent on time: $q = q(x,y,t) = A_1 t,$ where A_1 is loading speed.

Inflection of the "load-deflection" curve is the criterion for loss of stability of the shell under dynamic loading.

We shall consider cylindrical shell panels with length $a = 150h$, turning angle $b = 0.4$ rad., radius $R = 250h$ and thickness $h = 0.01$ m, with loading speed $A_1 = 1\,MPa/s$. The direction of the orthotropy axis 2 coincides with the direction of the generatrix (the x axis). Calculations were performed for $N = 9$.

Panels made of orthotropic materials are examined: E-Glass/Epoxy [4], AS/3501 Graphite/Epoxy [4], fiberglass T-10 / UPE22-27 [11], and carbon fiber-reinforced plastic (CFRP) M60J [12], as well as isotropic materials: steel and plexiglass. Material parameters and load values for loss of stability q_{kr} are given in Table 1, and the corresponding "load-deflection" curves in Figure 2.

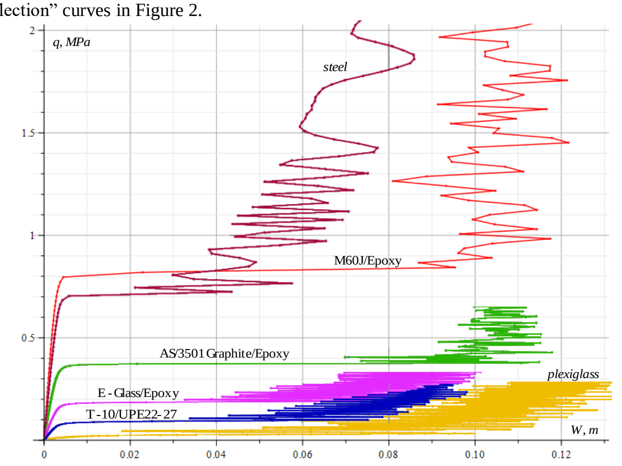

FIGURE 2. The "load-deflection" relations for the considered variants of shells

TABLE 1. Material parameters and load values for loss of stability q_{kr}

Material	E_1, MPa	E_2, MPa	G_{12}, MPa	μ_{12}	ρ, kg/m^3	q_{kr}, MPa
Steel (isotropic)	$2.1\cdot10^5$	$2.1\cdot10^5$	$0.807\cdot10^5$	0.3	7800	0.7032
Plexiglass (isotropic)	$0.03\cdot10^5$	$0.03\cdot10^5$	$0.012\cdot10^5$	0.35	1190	0.0284
E-Glass/Epoxy	$0.607\cdot10^5$	$0.248\cdot10^5$	$0.12\cdot10^5$	0.23	1800	0.1889
AS/3501 Graphite/Epoxy	$1.38\cdot10^5$	$0.0896\cdot10^5$	$0.071\cdot10^5$	0.3	1540	0.3721
Fiberglass T-10/UPE22-27	$0.294\cdot10^5$	$0.178\cdot10^5$	$0.0301\cdot10^5$	0.123	1800	0.0926
CFRP M60J	$3.3\cdot10^5$	$0.059\cdot10^5$	$0.039\cdot10^5$	0.32	1600	0.8194

CONCLUSIONS

Thus, the stability of some variants of cylindrical orthotropic panels made of modern orthotropic materials under dynamic loading was analyzed.

Based on the performed calculations, it can be concluded that when using modern orthotropic materials (carbon fiber-reinforced plastic, fiberglass, etc.) reduction of critical load value is possible, but such structures are substantially lighter than structures made of traditional isotropic materials (steel).